

The Monochord in Ancient Greek Harmonic Science

Among the many instruments devised by students of mathematical sciences in ancient Greece, the monochord provides one of the best opportunities to examine the methodologies of those who employed it in their investigations. Consisting of a single string which could be divided at measured points by means of movable bridges, it was used to demonstrate theorems about the arithmetical relationships between pitched sounds in music. This book traces the history of the monochord and its multiple uses down to Ptolemy, bringing together all the relevant evidence in one comprehensive study. By comparing the monochord with a number of other ancient scientific instruments and their uses, David Creese shows how the investigation of music in ancient Greece not only shares in the patterns of demonstrative and argumentative instrument use common to other sciences, but also goes beyond them in offering the possibility of a rigorous empiricism unparalleled in Greek science.

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CAMBRIDGE CLASSICAL STUDIES

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DAVID CREESE





> CAMBRIDGE UNIVERSITY PRESS Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi, Dubai, Tokyo

> > Cambridge University Press
> > The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org Information on this title: www.cambridge.org/9780521843249

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First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Creese, David E. (David Evan), 1972–

The monochord in ancient Greek harmonic science / David Creese.

p. cm. – (Cambridge classical studies)
 Includes bibliographical references and index.
 ISBN 978-0-521-84324-9 (hardback)

1. Monochord. 2. Musical intervals and scales – Greece – History – To 500.

3. Music theory – Greece – History – To 500. I. Title. II. Series. ML3809.c87 2009

787.7 – dc22 2009040103

ISBN 978-0-521-84324-9 Hardback

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CONTENTS

re	eface	page vii
Lis	t of abbreviations and editions	ix
Lis	t of figures	xiv
Lis	t of tables	xvi
	Introduction: The geometry of sound	I
Ι	Hearing numbers, seeing sounds: the role of	
	instruments and diagrams in Greek harmonic	
	science	22
	I Hearing numbers: arithmetic, geometry and	
	canonic division	23
	2 Seeing sounds: instruments, diagrams and tables	50
2	Mathematical harmonics before the monochord	81
	I The demonstration of the concord ratios	93
	2 The first appearance of the monochord	97
	3 Philolaus	104
	4 Archytas	117
3	The monochord in context	131
	1 Demonstration, harmonics and the Sectio canonis	136
	2 Finding a harmonic syntax: the 'ditonic' division	156
	3 Acoustics and the Sectio canonis	164
	4 The kanōn and the Sectio canonis	171
4	Eratosthenes	178
5	Canonic theory	210
-	The language of canonics	214
	2 The practice of canonics	226



CONTENTS

6 Ptolemy's canonics	283
Conclusion	356
Bibliography	360
Index locorum	374
General index	388



PREFACE

This book is about an instrument that shaped the way scientists thought about music in Greek antiquity. It is also about the way Greek science shaped the use of a musical instrument. With its single string, movable bridge and graduated rule, the monochord (kanōn) straddled the gap between notes and numbers, intervals and ratios, sense-perception and mathematical reason. By representing musical sounds as visible, measurable distances (lengths of string), and by representing numbers audibly to the musical ear, it offered a way to study music as an arithmetical science through the medium of geometry. As an audible extension of the lettered diagram of Greek mathematics, what it offered the harmonic scientist, most importantly, was a means of attempting to extend the irrefutability of mathematical argument into the auditory realm, even when whether or not it succeeded in doing so was controversial.

It gives me great pleasure to record my thanks to the many people who have helped me in the writing of this book. My greatest debt is to Andrew Barker, who supervised the doctoral thesis on which it is based, and whose continued interest, encouragement and critical engagement were immensely helpful not only during the initial research but also during the final revisions of the book, and at countless points in between. I also benefited greatly from the comments and advice of my doctoral examiners, Charles Burnett and Niall Livingstone, many of whose questions and ideas have caused me to rethink my interpretations of the ancient evidence and reshape my arguments about them in this book. G. E. R. Lloyd was especially generous in reading the entire manuscript twice in different versions and offering detailed and challenging suggestions for its improvement, particularly in the development of comparisons between harmonics and other ancient sciences. I am also grateful to Alan C. Bowen, Peter Merten, Nathan Sidoli and Malcolm

vii



PREFACE

Wilson for reading sections of the book in draft and for correcting some of my more egregious misunderstandings of astronomy and physics. For their invitations to present papers related to the research for this book, I should like to thank John Beatty, Sylvia Berryman, Robert Daum, Ingrid Holmberg, Donatella Restani and Gordon Shrimpton. Maya Yazigi answered my questions about the Arabic text of Diocles' *On Burning Mirrors*; Colin Gough helped me to understand the physics of sympathetic vibration; and Leonid Zhmud corresponded with me about Duris of Samos and the early history of the monochord. I have also benefited from discussions with Sylvia Berryman, Lisa Cooper, C. W. Marshall, Hallie Marshall, Francesco Pelosi, Massimo Raffa, Eleonora Rocconi and Robert B. Todd.

Final revisions to the manuscript were carried out during a period of research leave generously granted by the University of British Columbia, part of which was spent at the Scuola Normale Superiore in Pisa. I am especially grateful to Chiara Martinelli and Glenn Most for their invitation to spend what was a very productive time there, and also to Adrian Hasler for providing a similarly ideal (if very different) research environment during the previous summer. For their patience and assistance I thank Michael Sharp and Elizabeth Hanlon at Cambridge University Press, and for her care in copy-editing a demanding typescript I thank Linda Woodward, whose knowledge of the subject was as helpful as her attention to detail. I have received support of many kinds from my family, especially my parents, who read and commented on parts of the book, and above all my wife, Maggi, whose readings of many drafts saved me from errors too numerous to mention, and without whose constant interest, affection and assistance the book could not have been written at all

viii



ABBREVIATIONS AND EDITIONS

Ael. VH Aelianus, Varia Historia

Archim. Method Archimedes, Method of Mechanical

Problems

Archyt. Archytas, ed. Huffman 2005

Arist. Aristotle

An. post. Analytica posteriora, ed. Ross 1964

An. pr. Analytica priora

De an. De anima

Eth. Nic. Ethica Nicomachea

Gen. an. De generatione animalium

Metaph.MetaphysicaPh.PhysicaPol.PoliticaSens.De sensuTop.Topica[Arist.][Aristotle]

De audib. De audibilibus (ap. Ptol. *In Harm.*),

ed. Düring 1932

Pr. Problemata, ed. Hett 1936

Aristid. Quint. Aristides Quintilianus, *De musica*, ed.

Winnington-Ingram 1963, cited either by book and chapter (e.g. III.3) or by page

and line number (e.g. 99.9–12)

Aristox. Aristoxenus

El. harm. Elementa harmonica, ed. da Rios 1954

fr. Fragmenta, ed. Wehrli 1945 Ath. Athenaeus, *Deipnosophistae*

Bacchius, *Isagoge artis musicae*, ed. Jan

1895

Boeth. Mus. Boethius, De institutione musica, ed.

Friedlein 1867

ix



ABBREVIATIONS AND EDITIONS

Chrysippus, ed. von Arnim 1903–24

Cic. Att. Cicero, Epistulae ad Atticum

De audib.see [Arist.] De audib.Diog. Laert.Diogenes LaertiusDKDiels and Kranz 1951

Eratosth. Eratosthenes

Euc. El. Euclid, Elementa, ed. Heiberg 1883–8 [Euc.] Sect. can. [Euclid], Sectio canonis, ed. Jan 1895 Eutoc. In Sph. Cyl. Eutocius, On Archimedes' Sphere and

Cylinder, ed. Heiberg and Stamatis 1915

Exc. ex Nicom. Excerpta ex Nicomacho, ed. Jan 1895 Exc. Neap. Excerpta Neapolitana, ed. Jan 1895 FGrH Fragmente der Griechischen Historiker,

Jacoby 1923-58

FHG Fragmenta Historicorum Graecorum,

Müller 1841–70

Gal. De plac. Galen, De placitis Hippocratis et

Hipp. et Plat. Platonis

Gaud. Gaudentius, *Isagoge harmonica*, ed. Jan

1895, cited either by chapter (e.g. 11) or by page and line number (e.g. 341.13–17)

Gell. NA Aulus Gellius, Noctes Atticae Gem. Intro. astr. Geminus, Introductio astronomiae

Hdt. Herodotus

Heron of Alexandria

Deff. Definitiones, ed. Heiberg 1903

Stereom. Stereometrica
Hippoc. Hippocrates

Off. De officina medici

Vict. On Regimen, ed. Joly 1967

Hom. Homer

Il. Iliad

Od. Odyssey

Hor. Carm. Horace, Carmina

Iambl. Iamblichus

Comm. math. De communi mathematica scientia



ABBREVIATIONS AND EDITIONS

In Nic. In Nicomachi Arithmeticam

introductionem

VP De vita Pythagorica

Isoc. Isocrates

LSJ H. G. Liddell, and R. Scott, 1843, A

Greek–English Lexicon, 9th edn, ed. H. S. Jones (Oxford, 1940), with a revised supplement, ed. P. G. W. Glare (Oxford,

1996)

Macrob. In Somn. Macrobius, Commentarius ex Cicerone

in Somnium Scipionis

Nicom. Nicomachus of Gerasa

Ar. Arithmetica introductio

Harm. Harmonicum enchiridium, ed. Jan

1895, cited either by chapter (e.g. 6) or by page and line number (e.g.

246.1-5)

Papp. Pappus of Alexandria, *Collectio*, ed.

Hultsch 1876-8

PBerol. Berlin Papyri

PHerc. Papyri Herculanenses

Pherecrates Pherecrates

PHib. Hibeh Papyri, ed. Grenfell and Hunt

1906

Philo, Opif. Philo of Alexandria, De opificio mundi

Philolaus ed. Huffman 1993

Phld. *De mus*. Philodemus, *De musica* IV, ed. Delattre

2007

Pl. Plato

Euthd. Euthydemus Gorgias Grg. Lach. Laches Leges Leg. Meno Men. Phd. Phaedo Phdr. **Phaedrus** Phlb. Philebus



ABBREVIATIONS AND EDITIONS

Resp. Respublica, ed. Slings 2003
Ti. Timaeus, ed. Burnet 1902

[Pl.] Epin. [Plato] Epinomis

Plut. Plutarch Alc. Alcibiades

De an. procr.

Non posse

Non posse

Non posse suaviter vivi secundum

Epicurum, ed. Einarson and De Lacy

1967

Pericle Pericles

Quaest. conv. Quaestiones convivales Quaest. Plat. Quaestiones Platonicae

Sol. Solon

PMG Poetae Melici Graeci, Page 1962

Poll. Onom. Pollux, Onomasticon, ed. Bethe 1900–37

Porph. Porphyry

In Harm. In Ptolemaei Harmonica

commentarium, ed. Düring 1932

VP Vita Pythagorae POxy. Oxyrhynchus Papyri

Procl. Proclus

Hypotyp. Hypotyposis

In Euc. In primum Euclidis elementorum

librum commentarii

In Ti. In Platonis Timaeum commentaria,

ed. Diehl 1904

Ptol. Ptolemy

Alm. Almagest (= Syntaxis mathematica),

ed. Heiberg 1898–1903, cited either by book and chapter (e.g. V.1) or by page and line number (e.g. 351.12–19)

Harm. Harmonica, ed. Düring 1930, cited

either by book and chapter (e.g. I.10) or by page and line number (e.g.

23.21-24.1)

Judic. De iudicandi facultate et animi

principatu (Περὶ κριτηρίου καὶ

ήγεμονικοῦ), ed. Huby and Neal 1989

xii



ABBREVIATIONS AND EDITIONS

Sect. can. see [Euc.] Sect. can.

Sent. Vat. Vatican Sayings (= Gnomologium

Vaticanum)

Sext. Emp. Math. Sextus Empiricus, Adversus

mathematicos

Simpl. *In Phys.* Simplicius, *In Aristotelis physicorum*

libros commentaria, ed. Diels 1882-95

Stob. Flor. Stobaeus, Άνθολόγιον

Suppl. Hell. Supplementum Hellenisticum, ed.

Lloyd-Jones and Parsons 1983

SVF Stoicorum veterum fragmenta, ed. von

Arnim 1903–24

Theo. Sm. Theon of Smyrna, Expositio rerum

mathematicarum ad legendum Platonem

utilium, ed. Hiller 1878

Theophr. Theophrastus

Char. Characteres, ed. Diggle 2004

fr. Fragmenta, ed. Fortenbaugh et al. 1992

Thuc. Thucydides

Ti. Locr. [Timaeus Locrus], De universi natura

(Περὶ φύσιος κόσμω καὶ ψυχᾶς), ed. Thesleff 1965, cited by chapter and, where necessary, by page and line number; chapters 21–3 correspond to

96b–c

TLG Thesaurus Linguae Graecae,

http://www.tlg.uci.edu/



FIGURES

I.I	Diagram accompanying Sect. can. prop. 3.	page 25
I.2	Diagram accompanying Euc. El. VI.13.	37
1.3	Diagram based on that of Sect. can. prop. 3 (fig. 1.1)). 38
I.4	Diagram accompanying Papp. III. II.	39
1.5	Part of the diagram accompanying Sect. can.	
	prop. 20.	41
1.6	a-b. The <i>lēpsis dia symphōnias</i> in its simplest form	. 44
I.7	Geometrical canonic division of the epogdoic tone	
	into two equal semitones.	48
1.8	First diagram accompanying Ptol. Alm. V.2.	65
1.9	Diagram accompanying Ptol. Harm. I.8.	69
I.IO	Diagram accompanying Ptol. Harm. I.9.	73
2.I	The intervals of Philolaus testimonium A26 set out	
	on a <i>kanōn</i> .	117
4.I	The enharmonic and chromatic tetrachords of	
	Aristoxenus and Eratosthenes.	190
4.2	Eratosthenes' tetrachords as proposed by	
	Schlesinger.	194
4.3	Eratosthenes' mesolabon.	198
4.4	Five-panelled <i>mesolabon</i> arranged to divide an	
	epitritic fourth into five equal semitones.	199
5.I	Adrastus' canonic division applied to a 72-unit	
	kanōn.	246
5.2 a.	The thirty-six terms of Ti. Locr. 22 and the interval	S
	between them.	266
b.	The original seven terms of Pl. Ti. 35b-36b and the	;
	intervals between them.	
c.	Plato's seven terms set out in a Crantor-	
	(or 'lambda'-) diagram.	
5.3 a.	The division of <i>Ti. Locr.</i> 22.	276
b.	Thrasyllus' canonic division.	
xiv		



LIST OF FIGURES

6. I	Diagram accompanying Ptol. <i>Harm.</i> I.8 (= fig. 1.9).	307
6.2	An extension of the diagram accompanying the last	
	part of Ptol. Harm. III.2.	313
6.3	Diagram accompanying Ptol. Harm. I.10, 22.2–16.	321
6.4	Diagram accompanying Ptol. Harm. I.11, 26.3–14.	326
6.5	The <i>helikōn</i> instrument.	337
6.6	Ptolemy's variant on the <i>helikōn</i> .	339
6.7	Eratosthenes' $mesolabon$ (= fig. 4.3).	341
6.8	Diagram accompanying Ptol. Harm. III.9.	352
6.9	The circumference of Ptolemy's divided circle	
	(fig. 6.8), cut and laid flat as a kanōn.	353



TABLES

4. I	Nine of the tables of Ptol. <i>Harm</i> . II.14, showing		
•		oage	184
4.2	Comparison of rational and irrational conversions	_	
	of six Aristoxenian tetrachords into string lengths.		200
4.3	The ditonic diatonic as a suitable conversion of		
	the Aristoxenian tense diatonic, but not of the		
	soft diatonic.		204
4.4	The diatonic division-tables of Aristoxenus and		
	Eratosthenes at Ptol. Harm. II.14.		205
4.5	Number of panels needed to check Eratosthenean		
	conversion of Aristoxenian divisions on the		
	mesolahon		206

xvi